Automatic Control

Chapter four

Time domain analysis

By

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- It is important to test the control system after deriving it to cheek the behavior of the output (the desired) to the given input r(t) or R(s).
- The response of the output due to the given input with respect to time (i.e. varying of output with respect to time; t) is called time response.
- The time response of a control system (Y) is given as:

$$Y(t) = Y_t(t) + Y_{ss}(t) \quad \text{----- (4.1)}$$

Where:

Y_t is the transient response

Y_{ss} is the steady-state response

- The transient response will vanish after long period of time and the steady state response will reamin

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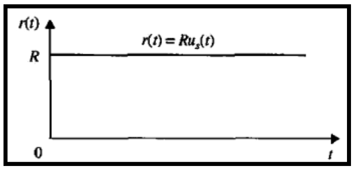
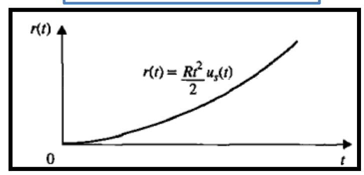
Test signals

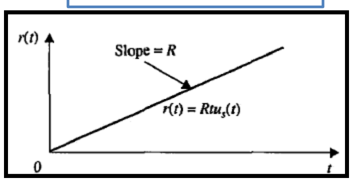
Unit step input



Parabolic function input



Ramp function input



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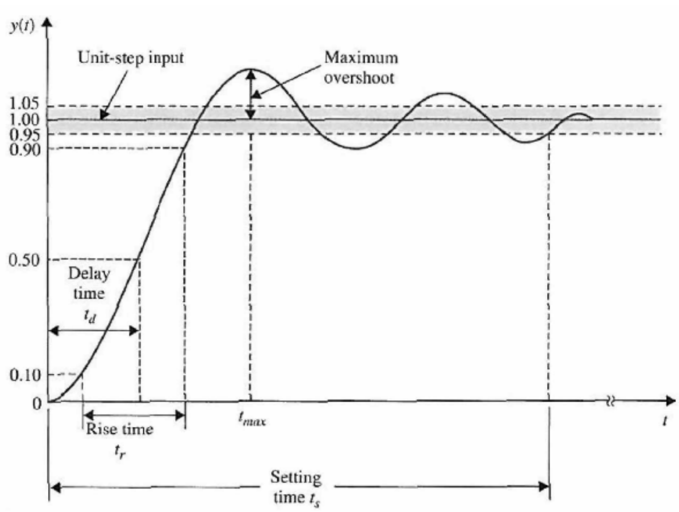
Time response for unit step input



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Example 4.1

However, consider a velocity control system in which a step input is used to control the system output that contains a ramp in the steady state. The system transfer functions may be of the form

$$G(s) = \frac{1}{s^2(s+12)} \qquad H(s) = K_t s$$

let *K, = 10 volts/rad/sec and* a unit-step input of 1 vol

Determine time response

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Example 4.1

Solution

For the system shown previously $M(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{1}{s(s^2+12s+10)}$

For a unit-step function input, $R(s) = l/s$. *then* $Y(s) =$

$$Y(s) = \frac{1}{s^2(s^2+12s+10)}$$

Take Laplace inverse to find the time domain response

$$y(t) = 0.1t - 0.12 - 0.000796e^{-11.1t} + 0.1208e^{-0.901t} \qquad t \geq 0$$

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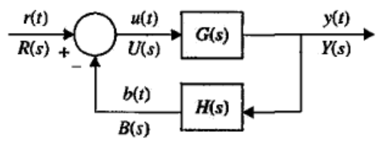
Steady state error (e_{ss})

Definition

Let us consider the feedback control system



$$e(t) = \text{reference signal} - y(t)$$

$$e(t) = r(t) - y(t)$$

The steady-state error is defined as

$$e_{ss} = \lim_{t\to\infty} e(t)$$

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Example 4.2

For the previous system discussed in example 4.1. find the steady state error.

Solution

The response (Y(s)) was found as:

$$y(t) = 0.1t - 0.12 - 0.000796e^{-11.1t} + 0.1208e^{-0.901t} \quad t \geq 0$$

Because the exponential terms of *y(t) in Eq. above all diminish as t —> ∞, the steady-state* part of *y(t) is 0.1t — 0.12. Thus, the steady-state error of the system is :*

$$e_{ss} = \lim_{t\to\infty}[0.1t - y(t)] = 0.12$$

Time domain analysis

Steady state error for unity feed back system

A u t o m a t i c C o n t r o l

To study the steady state error fro the three common test signals, we must review the representation of transfer function. one way of representing the transfer function is given as follow:

$$G(s) = \frac{K(1+T_1 s)(1+T_2 s)\cdots(1+T_{m1}s+T_{m2}s^2)}{s^j(1+T_a s)(1+T_b s)\cdots(1+T_{n1}s+T_{n2}s^2)}e^{-T_d s}$$

- where *K and all the T's are real constants*
- the system type refers to the order of the pole of *G(s) at s=0.*

Time domain analysis

Steady state error for unity feed back system

A u t o m a t i c C o n t r o l

Unity step input $R(s) = R/s$

$$e_{ss} = \lim_{s\to 0}\frac{sR(s)}{1+G(s)} = \lim_{s\to 0}\frac{R}{1+G(s)} = \frac{R}{1+\lim_{s\to 0}G(s)}$$

For convenience, we define

$$K_p = \lim_{s\to 0} G(s)$$

as the step-error constant. Then

$$e_{ss} = \frac{R}{1+K_p}$$

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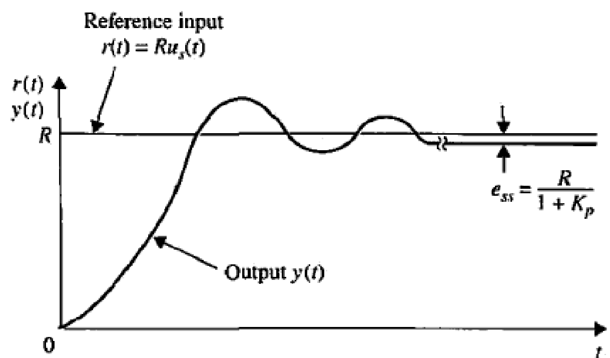
Steady state error for unity feed back system

Unity step input $R(s) = R/s$



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Steady state error for unity feed back system

Ramp input $R(s) = \frac{R}{s^2}$

$$e_{ss} = \lim_{s \to 0} \frac{R}{s + sG(s)} = \frac{R}{\lim_{s \to 0} sG(s)}$$

We define the ramp-error constant as

$$K_v = \lim_{s \to 0} sG(s)$$

Then,

$$e_{ss} = \frac{R}{K_v}$$

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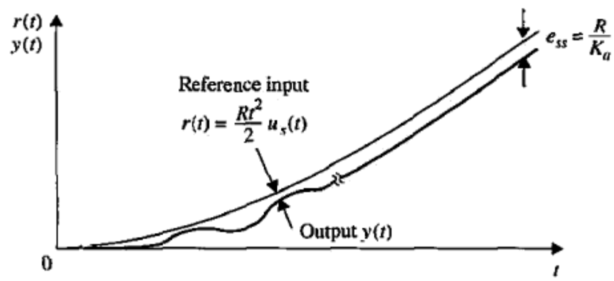
Steady state error for unity feed back system

Parabolic input $R(s) = \frac{R}{s^3}$

$$e_{ss} = \frac{R}{\lim_{s \to 0} s^2 G(s)} \qquad K_a = \lim_{s \to 0} s^2 G(s) \qquad e_{ss} = \frac{R}{K_a}$$



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Steady state error for unity feed back system

TABLE 5-1 Summary of the Steady-State Errors Due to Step-, Ramp-, and Parabolic-Function Inputs for Unity-Feedback Systems

Type of System	Error Constants			Steady-State Error e_{ss}: Step Input	Ramp Input	Parabolic
j	K_p	K_v	K_a	$\frac{R}{1+K_p}$	$\frac{R}{K_v}$	$\frac{R}{K_a}$
0	K	0	0	$\frac{R}{1+K}$	∞	∞
1	∞	K	0	0	$\frac{R}{K}$	∞
2	∞	∞	K	0	0	$\frac{R}{K}$
3	∞	∞	∞	0	0	0